Ex.18　解: 实对称矩阵A 可对角化, 与特征值-1 对应的特征向量为$\xi_1=(0,1,1)^T$, 假设属于特征值1 的特征向量为x, 则有

$$(0,1,1)x=0\Rightarrow x_2=0\cdot x_1-x_3.$$

自由变量x_1,x_3 取值

$$\begin{pmatrix}x_1\\x_3\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix},\quad \begin{pmatrix}x_1\\x_3\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}.$$

基础解系为

$$\xi_2=\begin{pmatrix}1\\0\\0\end{pmatrix},\quad \xi_3=\begin{pmatrix}0\\-1\\1\end{pmatrix}.$$

显然, 向量ξ_1,ξ_2,ξ_3 两两正交, 只需单位化, 令

$$\eta_1=\frac{\xi_1}{\sqrt{(\xi_1,\xi_1)}}=\begin{pmatrix}0\\\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix},\ \eta_2=\frac{\xi_2}{\sqrt{(\xi_2,\xi_2)}}=\begin{pmatrix}1\\0\\0\end{pmatrix},\ \eta_3=\frac{\xi_3}{\sqrt{(\xi_3,\xi_3)}}=\begin{pmatrix}0\\-\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}.$$

令

$$P=\begin{pmatrix}0&1&0\\\frac{1}{\sqrt{2}}&0&-\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}&0&\frac{1}{\sqrt{2}}\end{pmatrix},\quad \Lambda=\begin{pmatrix}-1&&\\&1&\\&&1\end{pmatrix}.$$

于是,

$$A=P\Lambda P^{-1}=P\Lambda P^T=\begin{pmatrix}1&0&0\\0&0&-1\\0&-1&0\end{pmatrix}.$$